Machine Learning for Robotics SS 2017
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Exercise Sheet 1
April 20, 2017

1 Refresher on Linear Algebra and Derivatives

(a) Let A be a 3×4 matrix and B a 3×2 matrix, what is the size of $A^T B$.

(b) Let $x \in \mathbb{R}^n$ be a column vector (vectors are always columns for us) and A a $m \times n$ matrix. What is the size of Ax.

(c) What is the derivative of $f(x) = (2x + y)^2$ w.r.t. x: $\frac{\partial}{\partial x} f(x)$?

(d) Given $f(x) = g(x^2)$ where $g(x) = (x + y)^2$, what is $\frac{\partial}{\partial x} f(x)$?

2 Multivariable Calculus

Recall that a matrix $A \in \mathbb{R}^{n \times n}$ is *symmetric* if $A^T = A$, that is, $A_{ij} = A_{ji}$ for all i, j. Also recall the gradient $\nabla f(x)$ of a function $f : R^n \to R$ is the $n-$vector of partial derivatives

$$\nabla f(x) = \begin{pmatrix} \frac{\partial}{\partial x_1} f(x) \\ \vdots \\ \frac{\partial}{\partial x_n} f(x) \end{pmatrix} \qquad \text{where } x = \begin{pmatrix} x1 \\ \vdots \\ x_n \end{pmatrix}.$$

The hessian $\nabla^2 f(x)$ is the $n \times n$ symmetric matrix of twice partial derivatives,

$$\nabla^2 f(x) = \begin{pmatrix} \frac{\partial^2}{\partial x_1^2} f(x) & \frac{\partial^2}{\partial x_1 \delta x_2} f(x) & \cdots & \frac{\partial^2}{\partial x_1^2 \delta x_n} f(x) \\ \frac{\partial^2}{\partial x_2 \delta x_1} f(x) & \frac{\partial^2}{\partial x_2^2} f(x) & \cdots & \frac{\partial^2}{\partial x_2 \delta x_n} f(x) \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial^2}{\partial x_n \delta x_1} f(x) & \frac{\partial^2}{\partial x_n \delta x_2} f(x) & \cdots & \frac{\partial^2}{\partial x_n^2} f(x) \end{pmatrix}.$$

(a) Let $f(x) = \frac{1}{2} x^T A x + b^T x$, where A is a symmetric matrix and $b \in \mathbb{R}^n$ is a vector. What is $\nabla f(x)$?

(b) Let $f(x) = g(h(x))$, where $g : \mathbb{R} \to \mathbb{R}$ is differentiable and $h : \mathbb{R}^n \to \mathbb{R}$ is differentiable. What is $\nabla f(x)$?

(c) Let $f(x) = \frac{1}{2} x^T A x + b^T x$ as in a. What is $\nabla^2 f(x)$?

(d) Let $f(x) = g(a^T x)$, where $g : \mathbb{R} \to \mathbb{R}$ is continuously differentiable and $a \in R^n$ is a vector. What are $\nabla f(x)$ and $\nabla^2 f(x)$?

3 Hands On

The goal of this exercise is twofold. First, you should get familiar with Python and second, you should get hands-on experience with regularization and model selection. Download the `diabetes.txt` dataset from the course website. Use the first 10 columns as features and the last column as target value.

I highly recommend to use IPython and Jupyter notebooks (just google for it) we will use it later as well.

- You need the `numpy` and `matplotlib` packages

  ```
  import numpy as np
  import matplotlib.pyplot as plt
  import csv
  %matplotlib inline
  ```

  read the file

  ```
  with open("diabetes.txt","rb") as file:
    reader = csv.reader(file, delimiter=' ')
    table = np.asarray([row for row in reader], dtype=np.float)
  ```

  Read about numpy and how to manipulate matrices etc.

- run the following experiment by taking the first 200 data points as training and the next 200 points as validation set
- train a least squares regression model without regularization (using matrix operations)
- for each regularization parameters $\lambda \in \{2^{-20}, 2^{-19}, \ldots, 2^{10}\}$ train a ridge regression model with regularization λ (using matrix operations)
- evaluate the trained models on the validation set
- plot the results in a style that you find most appropriate/informative including the selected regularization parameter.