Machine Learning for Robotics SS 2017
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Due-date: 11.05.2017 (hand in at beginning of recitation session)
Exercise Sheet 3
May 8, 2017

1 Bayes Classifier

In the lecture we saw that the Bayes classifier is

$$c^*(x) := \arg\max_{y\in\mathcal{Y}} p(y|x). \tag{1}$$

a) Which of these decision functions is equivalent to c^*?

- $c_1(x) := \arg\max_y p(x)$
- $c_2(x) := \arg\max_y p(y)$
- $c_3(x) := \arg\max_y p(x, y)$
- $c_4(x) := \arg\max_y p(x|y)$

For $\mathcal{Y} = \{-1, +1\}$, we can express the Bayes classifier as $c^*(x) = \text{sign}[\log \frac{p(+1|x)}{p(-1|x)}]$

b) Which of the following expressions are equivalent to c^*?

- $c_5(x) := \text{sign}[\frac{\log p(x,+1)}{\log p(x,-1)}]$
- $c_6(x) := \text{sign}[\log p(+1|x) + \log p(-1|x)]$
- $c_7(x) := \text{sign}[\log p(+1|x) - \log p(-1|x)]$
- $c_8(x) := \text{sign}[\log p(x, +1) - \log p(x, -1)]$
- $c_9(x) := \text{sign}[p(+1|x) - p(-1|x)]$
- $c_{10}(x) := \text{sign}[\frac{p(x,+1)}{p(x,-1)} - 1]$
- $c_{11}(x) := \text{sign}[\frac{\log p(+1|x)}{\log p(-1|x)} - 1]$
- $c_{12}(x) := \text{sign}[\log \frac{p(x|+1)}{p(x|-1)} + \log \frac{p(+1)}{p(-1)}]$

2 Logistic Regression

Visualize the cost term of logistic regression in 2D ($w \in \mathbb{R}^2$) for the following data points: $D = \{([-2, 1], 1), ([1, 0], 1), ([-1, -1], 1), ([0.1, 0.1], -1)\}$ which are the (x, y) pairs. Use a contourplot to show the iso-cost lines.

3 Proof that logistic regression is a convex optimization problem

In the lecture didn't show that the Hessian of the loss function of logistic regression is positive definite.

The cost term is:

$$\mathcal{L}(w) = \sum_{i=1}^{n} \log\left(1 + \exp(-y^i \langle w, x^i \rangle)\right) \tag{2}$$

Show that its Hessian is indeed a positive definite matrix.

Hint: compute $H_w \mathcal{L}(w) = \nabla \nabla^T \mathcal{L}(w)$ and use that $\hat{p}(y|x; w) = \frac{1}{1+\exp(-y\langle w, x \rangle)}$, $\mathcal{Y} = \{-1, 1\}$, and $\frac{1}{1+\exp(-z)} = \frac{\exp(z)}{1+\exp(z)}$.