Machine Learning for Robotics
Intelligent Systems Series
Lecture 6

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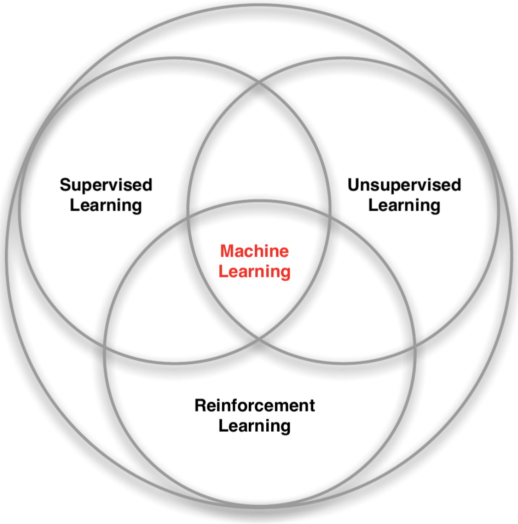
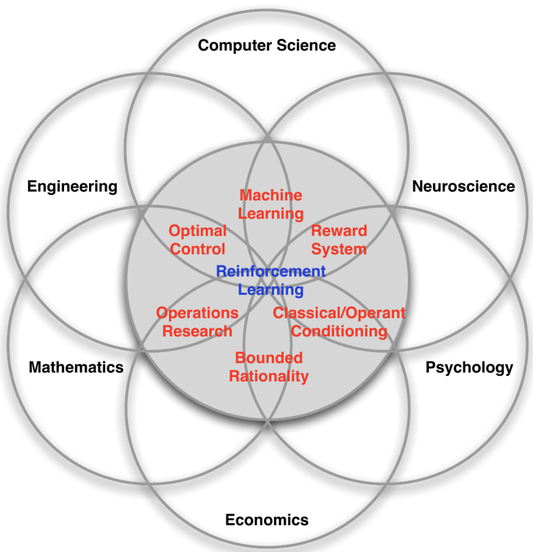
MAX-PLANCK-GESELLSCHAFT

Reminder: Branches of Machhine Learning



Many Types and Areas of Reinforcement Learning



Characteristics of Reinforcement Learning

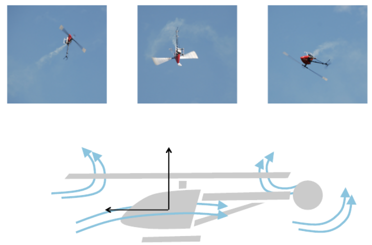
What makes reinforcement learning different from other machine learning paradigms?

- There is no supervisor, only a reward signal
- Feedback is delayed, not instantaneous
- Time really matters (sequential, non i.i.d data)
- Agent's actions affect the subsequent data it receives

Examples of Reinforcement Learning

- Fly stunt manoeuvres in a helicopter
- Defeat the world champion at Backgammon
- Manage an investment portfolio
- Control a power station
- Make a humanoid robot walk
- Play many different Atari games better than humans
- Beat the best human player in Go

Examples – Helicopter Manoeuvres



https://www.youtube.com/watch?v=0JL04JJjocc

Examples – Bipedal Robots

https://www.youtube.com/watch?v=No-JwwPbSLA

Examples – Atari Games

https://www.youtube.com/watch?v=Vr5MR5lKOc8

Rewards

- A reward R_t is a scalar feedback signal
- Indicates how well agent is doing at step t
- The agent's job is to maximize cumulative reward

Reinforcement learning is based on the reward hypothesis

Definition (Reward Hypothesis)

All goals can be described by the maximization of expected cumulative reward

Do you agree with this statement?

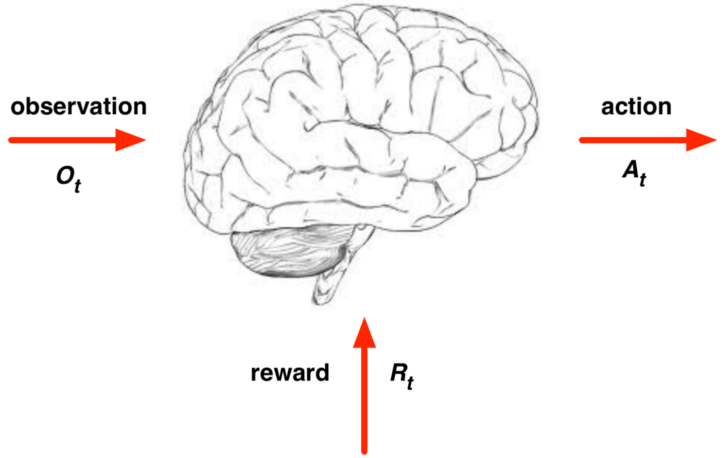
Examples of Rewards

- Fly stunt manoeuvres in a helicopter
  - +ve reward for following desired trajectory
  - -ve reward for crashing
- Defeat the world champion at Backgammon
  - +/-ve reward for winning/losing a game
- Manage an investment portfolio
  - +ve reward for each $ in bank
- Control a power station
  - +ve reward for producing power
  - -ve reward for exceeding safety thresholds
- Make a humanoid robot walk
  - +ve reward for forward motion
  - -ve reward for falling over
- Play many different Atari games better than humans
  - +/-ve reward for increasing/decreasing score

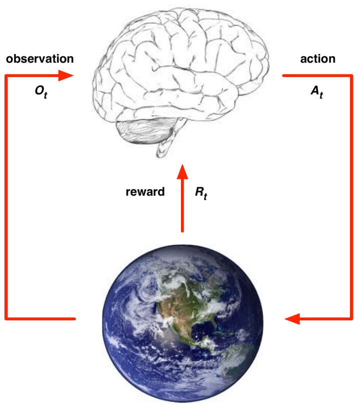
Sequential Decision Making

- Goal: select actions to maximize total future reward
- Actions may have long term consequences
- Reward may be delayed
- It may be better to sacrifice immediate reward to gain more long-term reward
- Examples:
  - A financial investment (may take months to mature)
  - Refueling a helicopter (might prevent a crash in several hours)
  - Blocking opponent moves (might help winning chances many moves from now)

Agent and Environment



Agent and Environment



- At each step t the agent:
  - Executes action A_t
  - Receives observation O_t
  - Receives scalar reward R_t
- The environment:
  - Receives action A_t
  - Emits observation O_{t+1}
  - Emits scalar reward R_{t+1}
- t increments at env. step

History and State

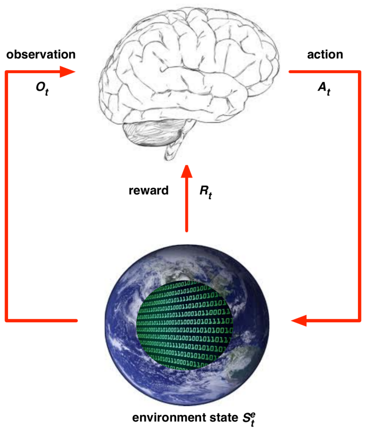
- The history is the sequence of observations, actions, rewards

$$H_t = O_1, R_1, A_1, ..., A_{t-1}, O_t, R_t$$

- i.e. all observable variables up to time t
- i.e. the sensorimotor stream of a robot or embodied agent
- What happens next depends on the history:
  - The agent selects actions
  - The environment selects observations/rewards
- State is the information used to determine what happens next
- Formally, state is a function of the history:

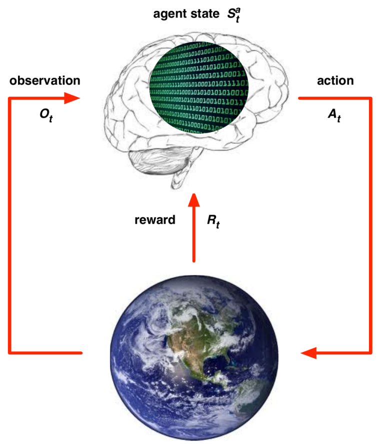
$$S_t = f(H_t)$$

Environment/World State



- The environment state S_t^e is the environment's private representation
- i.e. whatever data the environment uses to pick the next observation/reward
- The environment state is not usually visible to the agent
- Even if S_t^e is visible, it may contain irrelevant information

Agent State



- The agent state S_t^a is the agent's internal representation
- i.e. whatever information the agent uses to pick the next action
- i.e. it is the information used by reinforcement learning algorithms
- It can be any function of the history:

$$S_t^a = f(H_t)$$

Information State

An information state (a.k.a. Markov state) contains all useful information from the history.

Definition
A state S_t is Markov if and only if

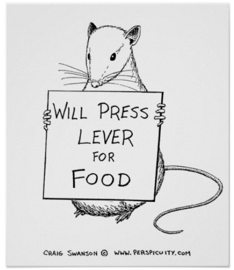
$$P(S_{t+1} \mid S_t) = P(S_{t+1} \mid S_1, ..., S_t)$$

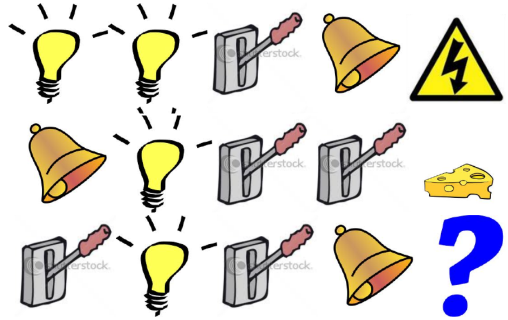
- The future is independent of the past given the present

$$H_{1:t} \rightarrow S_t \rightarrow H_{t+1:\infty}$$

- Once the state is known, the history may be thrown away
- i.e. the state is a sufficient statistic of the future
- The environment state S_t^e is Markov
- The history H_t is Markov

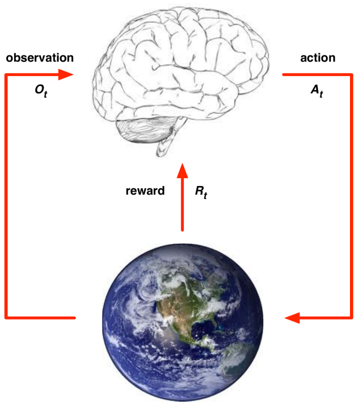
Rat Example





- What if agent state = last 3 items in sequence?
- What if agent state = counts for lights, bells and levers?
- What if agent state = complete sequence?

Fully Observeable Environments



Full observability: agent directly observes environment state

$$O_t = S_t^a = S_t^e$$

- Agent state = environment state = information state
- Formally, this is a Markov decision process (MDP)

Partially Observable Environments

- Partial observability: agent indirectly observes environment:
  - A robot with camera vision isn't told its absolute location
  - A trading agent only observes current prices
  - A poker playing agent only observes public cards
- Now agent state $\neq$ environment state
- Formally this is a partially observable Markov decision process (POMDP)
- Agent must construct its own state representation S_t^a, e.g.
  - Complete history: $S_t^a = H_t$
  - Beliefs of environment state: $S_t^a = (P(S_t^e = s^1), ..., P(S_t^e = s^n))$
  - Recurrent neural network: $S_t^a = \sigma(S_{t-1}^a W_s + O_t W_o)$

Major Components of an RL Agent

An RL agent may include one or more of these components:

- Policy: agent's behaviour function
- Value function: how good is each state and/or action
- Model: agent's representation of the environment

Policy

- A policy defines the agent's behaviour
- It is a map from state to action, e.g.
- Deterministic policy: $a = \pi(s)$
- Stochastic policy: $\pi(a|s) = P(A_t = a | S_t = s)$

Value Function

- Value function is a prediction of future reward
- Used to evaluate the goodness/badness of states
- ➡ used to select which action to take
- e.g. models the expected discounted future reward

$$v_\pi(s) = \mathbb{E}_\pi[R_{t+1} + \gamma R_{t+2} + \gamma R_{t+3} + \cdots \mid S_t = s]$$

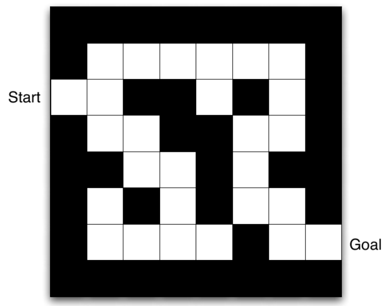
Model

- A model predicts what the environment will do next
- $\mathcal{P}$ predicts the next state
- $\mathcal{R}$ predicts the next (immediate) reward, e.g.

$$\begin{aligned} \mathcal{P}^a_{ss'} &= P(S_{t+1} = s' \mid S_t = s, A_t = a) \\ \mathcal{R}^a_s &= \mathbb{E}[R_{t+1} \mid S_t = s, A_t = a] \end{aligned}$$

- Can be used for planning without actually performing actions

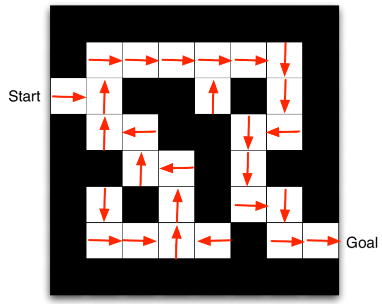
Maze Example



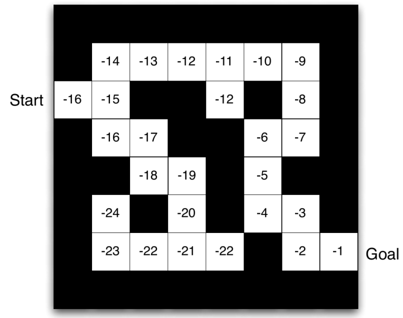
Environment

- Rewards: -1 per time-step
- Actions: N, E, S, W
- States: Agent's location
- End at Goal state

Maze Example: Policy and Value Function

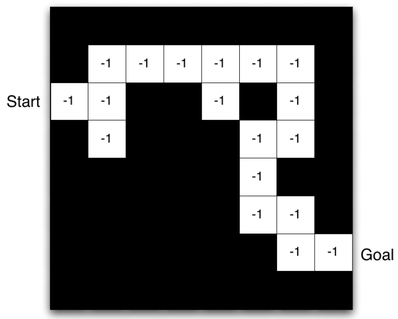


Arrows represent policy $\pi(s)$



Numbers represent value $v_\pi(s)$

Maze Example: Model



- Agent may have an internal model of the environment
- Dynamics: how actions change the state
- Rewards: how much reward from each state
- The model may be imperfect (most likely is)

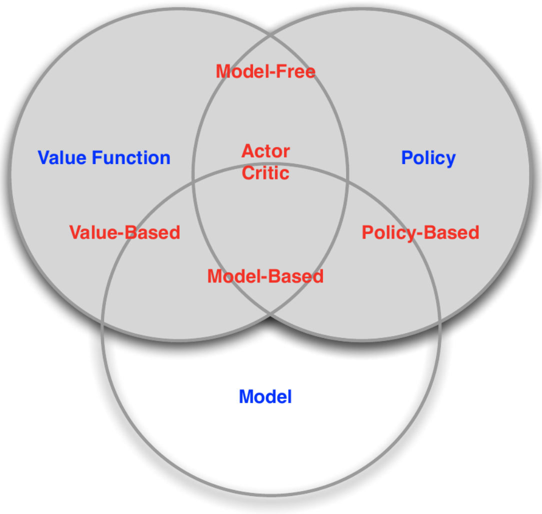
- Grid layout represents transition model $\mathcal{P}^a_{ss'}$
- Numbers represent immediate reward $\mathcal{R}^a_s$ from each state s (same for all a)

Categorization of RL agents

- Value Based
  - No Policy (Implicit)
  - Value Function
- Policy Based
  - Policy
  - No Value Function
- Actor Critic
- Policy
- Value Function

- Model Free
  - Policy and/or Value Function
  - No Model
- Model Based
  - Policy and/or Value Function
  - Model

RL Agent Taxonomy

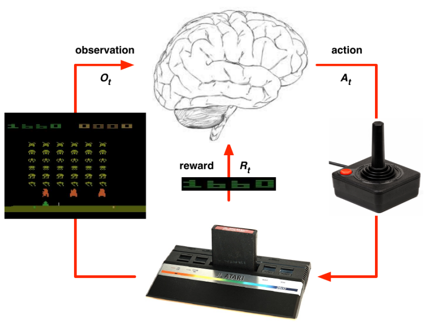


Learning and Planning

Two fundamental problems in sequential decision making

- Reinforcement Learning:
  - The environment is initially unknown
  - The agent interacts with the environment
  - The agent improves its policy
- Planning:
  - A model of the environment is known
  - The agent performs computations with its model (without any external interaction)
  - The agent improves its policy
  - a.k.a. deliberation, reasoning, introspection, pondering, thought, search

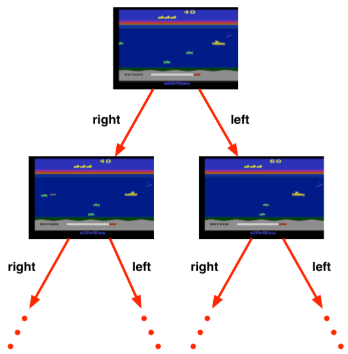
Atari Example: Reinforcement Learning



- Rules of the game are unknown
- Learn directly from interactive game-play
- Pick actions on joystick, see pixels and scores

Atari Example: Planning

- Rules of the game are known
- Can query emulator
  perfect model inside agent's brain
- If I take action a from state s:
  - what would the next state be?
  - what would the score be?
- Plan ahead to find optimal policy
  e.g. tree search



Exploration and Exploitation

- Reinforcement learning is like trial-and-error learning
- The agent should discover a good policy
- From its experiences of the environment
- Without losing too much reward along the way

- Exploration finds more information about the environment
- Exploitation exploits known information to maximize reward
- It is usually important to explore as well as exploit

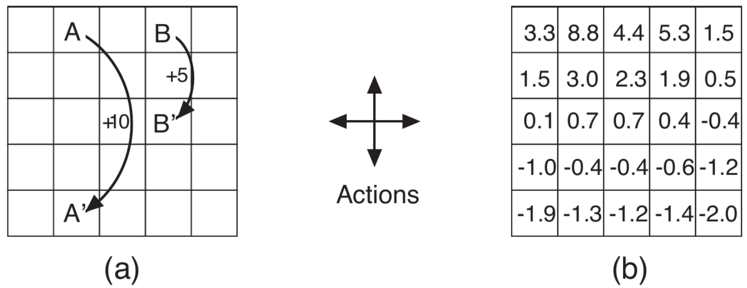
Examples

- Restaurant Selection
  Exploitation Go to your favorite restaurant
  Exploration Try a new restaurant
- Online Banner Advertisements
  Exploitation Show the most successful advert
  Exploration Show a different advert
- Game Playing
  Exploitation Play the move you believe is best
  Exploration Play an experimental move
- Robot Control
  Exploitation Do the movement you know works best
  Exploration Try a different movement

Prediction and Control

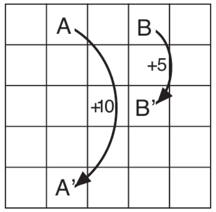
- Prediction: evaluate the future
  How do I do given a policy?
- Control: optimize the future
  Find the best policy

Gridworld Example: Prediction



- What is the value function for the uniform random policy?

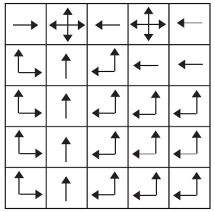
Gridworld Example: Control



a) gridworld

22.0	24.4	22.0	19.4	17.5
19.8	22.0	19.8	17.8	16.0
17.8	19.8	17.8	16.0	14.4
16.0	17.8	16.0	14.4	13.0
14.4	16.0	14.4	13.0	11.7

b) v_*



c) π_*

- What is the optimal value function over all possible policies?
- What is the optimal policy?

Markov Decision Processes

Markov Process

A Markov process is a memoryless random process, i.e. a sequence of random states $S_1, S_2, \ldots$ with the Markov property.

Reminder: Markov property

A state S_t is Markov if and only if

$$P(S_{t+1} \mid S_t) = P(S_{t+1} \mid S_1, \ldots, S_t)$$

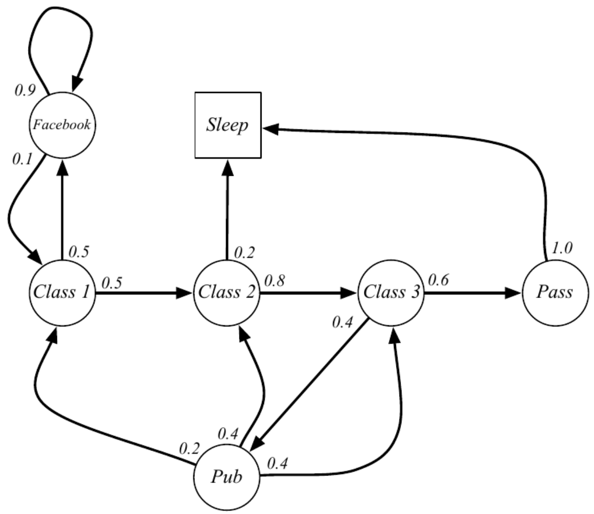
Definition (Markov Process/ Markov Chain)

A *Markov Process* (or *Markov Chain*) is a tuple $(\mathcal{S}, \mathcal{P})$

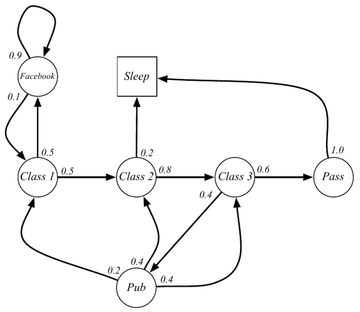
- $\mathcal{S}$ is a (finite) set of states
- $\mathcal{P}$ is a state transition 0 probability matrix,

$$P_{ss'} = P(S_{t+1} = s' \mid S_t = s)$$

Example: Student Markov Chain



Example: Student Markov Chain Episodes

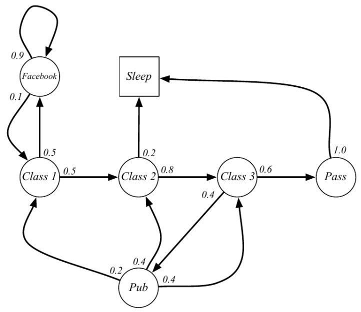


Sample episodes for starting from $S_1 =$C1

$$S_1, S_2, \ldots, S_T$$

- C1 C2 C3 Pass Sleep
- C1 FB FB C1 C2 Sleep
- C1 C2 C3 Pub C2 C3 Pass Sleep
- C1 FB FB C1 C2 C3 Pub C1 FB FB FB C1 C2 C3 Pub C2 Sleep

Example: Student Markov Chain Transition Matrix



$$\mathcal{P} = \begin{array}{c} \\ \textit{C1} \\ \textit{C2} \\ \textit{C3} \\ \textit{Pass} \\ \textit{Pub} \\ \textit{FB} \\ \textit{Sleep} \end{array} \begin{array}{c} \begin{array}{ccccccc} \textit{C1} & \textit{C2} & \textit{C3} & \textit{Pass} & \textit{Pub} & \textit{FB} & \textit{Sleep} \end{array} \\ \left[\begin{array}{ccccccc} & 0.5 & & & & 0.5 & \\ & & 0.8 & & & & 0.2 \\ & & & 0.6 & 0.4 & & \\ & & & & & & 1.0 \\ 0.2 & 0.4 & 0.4 & & & & \\ 0.1 & & & & & 0.9 & \\ & & & & & & 1 \end{array}\right] \end{array}$$

Markov Reward Process

A Markov reward process is a Markov chain with values.

Definition (MRP)

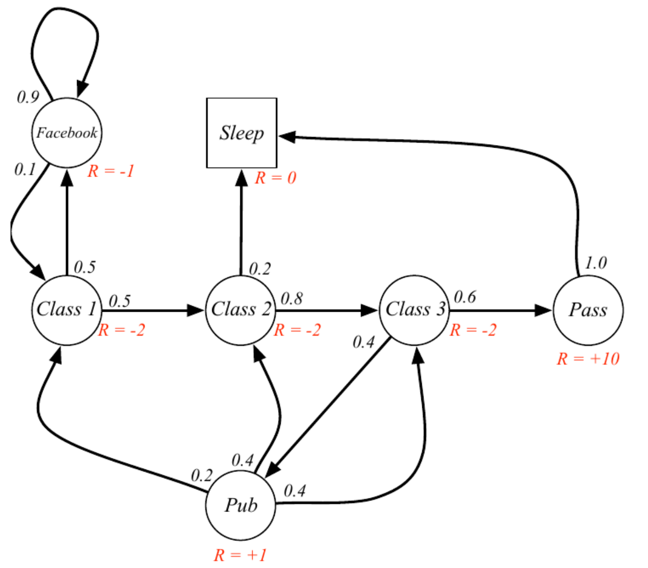
A *Markov Reward Process* is a tuple $(\mathcal{S}, \mathcal{P}, \mathcal{R}, \gamma)$

- $\mathcal{S}$ is a finite set of states
- $\mathcal{P}$ is a state transition probability matrix,

$$P(S_{t+1} \mid S_t) = P(S_{t+1} \mid S_1, \ldots, S_t)$$

- $\mathcal{R}$ is a reward function, $\mathcal{R}_s = \mathbb{E}[R_{t+1} | S_t = s]$
- γ is a discount factor, $\gamma \in [0, 1]$

Example: Student MRP



Return

Definition
The *return* G_t is the total discounted reward from time-step t.

$$G_t = R_{t+1} + \gamma R_{t+2} + \cdots = \sum_{k=0}^{\infty} \gamma^k R_{t+k+1}$$

- The discount $\gamma \in [0, 1]$ is the present value of future rewards
- The value of receiving reward R after $k + 1$ time-steps is $\gamma^k R$.
- This values immediate reward above delayed reward.
  - γ close to 0 leads to "myopic" evaluation
  - γ close to 1 leads to "far-sighted" evaluation

Why discount?

Most Markov reward and decision processes are discounted. Why?

- Mathematically convenient to discount rewards
- Avoids infinite returns in cyclic Markov processes
- Uncertainty about the future may not be fully represented
- If the reward is financial, immediate rewards may earn more interest than delayed rewards
- Animal/human behaviour shows preference for immediate reward
- It is sometimes possible to use *undiscounted* Markov reward processes (i.e. $\gamma = 1$), e.g. if all sequences terminate.

Next time

- Continue with MDP's and Bellmann Equation
- Dynamic Programming and Q-Learning